

# Thermal analysis of a building brick containing phase change material

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## Abstract

This paper presents the thermal analysis of a building brick containing phase change material (PCM) to be used in hot climates. The objective of using the PCM is to utilize its high latent heat of fusion to reduce the heat gain by absorbing the heat in the bricks through the melting process before it reaches the indoor space. The considered model consists of bricks with cylindrical holes filled with PCM. The problem is solved in a two-dimensional space using the finite element method. The thermal effectiveness of the proposed brick-PCM system is evaluated by comparing the heat flux at the indoor surface to a wall without the PCM during typical working hours. A parametric study is conducted to assess the effect of different design parameters, such as the PCM's quantity, type, and location in the brick. The results indicate that the heat gain is significantly reduced when the PCM is incorporated into the brick, and increasing the quantity of the PCM has a positive effect. PCM cylinders located at the centerline of the bricks shows the best performance.

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*Keywords:* Thermal insulation; PCM; Latent heat; Bricks

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## 1. Introduction

In hot climates, the thermal insulation materials are extensively used in building structures to reduce the heat flow into the building indoor space by providing an effective thermal resistance to the heat flow. In Kuwait, a study from the Kuwait ministry of energy [1] indicated that 70% of the produced electrical power in steam power plants is consumed by HVAC systems during the summer season, May to October, leading to high cooling bills. Fig. 1 shows the percentage of the monthly energy consumption by the HVAC systems to total energy produced in Kuwait. Hence, the importance of the insulation materials motivates the heat transfer engineers to additionally enhance the current technology of the insulation materials used in buildings. The commonly used insulation materials are fiberglass, cotton, and foams. Fiberglass insulation is soft wool-like material, made from sand and inorganic material, and it is originally used as a safe substitute for asbestos. In addition to insulation materials, the design of the building envelope highly influences the building's energy consumption. Minimizing window areas is recommended to minimize the solar radiation [2].

It is well known that the thermophysical properties of the insulation material have a strong effect on the heating and cooling energy consumptions. The thermal conductivity of the insulations is the property that affects the heat flow at a steady state condition. For an unsteady condition, the specific heat provides an additional thermal resistance to the heat flow, but it is ineffective when the system reaches the steady state condition. The variations of the solar intensity and outdoor temperature is time depended, and therefore, both the thermal conductivity and the specific heat of the insulation affect the heat flow. Insulation materials with high thermal capacity and low thermal conductivity are preferred. Over the past several decades, the integration of the phase change material (PCM) into building materials has been investigated as a potential technology for reducing the cooling and heating loads in building. The PCM insulation can be classified as a capacitive type of insulations because they slow down the heat flow by absorbing the heat. During daytimes, the PCM absorb part of the heat through the melting process, and at night, the PCM solidify and releases the stored heat. The net effect is a reduction of heat flow from outdoor to indoor space. PCMs are organic or inorganic substances with low melting temperature and high latent heat of fusion, such as paraffin and salt. During the melting process, the specific heat of the PCM increases to more than 100 times to absorb large quantity of energy, and during the solidification process, the stored energy is released.

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### Nomenclature

$C$	specific heat (kJ/kg °C)
$h$	heat transfer coefficient (W/m <sup>2</sup> °C)
$H$	height of the brick (m)
$H_c$	distance from the indoor surface to the center of the PCM cylinders (m)
$k$	thermal conductivity (W/m °C)
$n$	normal coordinate for a surface
PCM	phase change material
$q, \bar{q}$	heat flux, and average heat flux (W/m <sup>2</sup> )
$Q$	heat flow (W)
$t$	time (s)
$T$	temperature (°C)
$\Delta T$	phase change transition temperature (°C)
$W$	width of the computational domain (m)

### Greek symbols

$\alpha$	absorptivity
$\varepsilon$	emissivity
$\lambda$	latent heat (kJ/kg)
$\rho$	density (kg/m <sup>3</sup> )

### Subscripts

b	brick
conv	convection
correc	correction
i	indoor
is	indoor wall surface
m	melting
o	outdoor
os	outdoor wall surface
s	solar-air
surr	surrounding

### Superscript

$m$	iteration number
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The selection of the PCM is mainly based on the PCM's melting temperature. The PCM melting temperature should be within the operating temperature of the thermal system. Low cost, non-toxic, non-flammable, and chemically stable are preferred PCMs.

Using the PCM in building structure components to conserve energy was pointed out already in 1975 by Barkmann and Wessling [3], and later by Morikama et al. [4]. Ismail and Castro [5] enclosed a PCM layer between the external and internal walls. The outdoor surface was subjected to a time depended solar radiation while the indoor surface was kept at a constant temperature. The result indicated that the PCM was effective in maintaining the indoor temperature close to the established comfort limit. A PCM system consists of sodium thiosulfate pentahydrate absorbed into a porous concrete was investigated by Hadjienna et al. [6]. Stritih [7] numerically investigated heat storage device with fins to

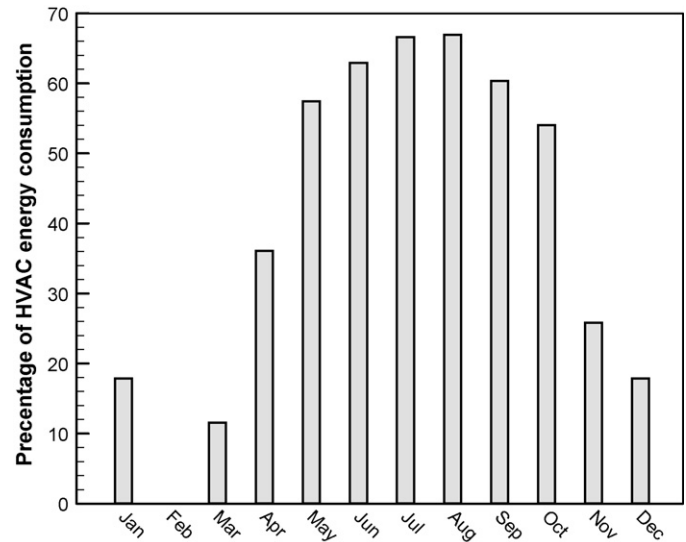


Fig. 1. The percentage of energy consumption by HVAC systems to the total produced energy in Kuwait for year 2002.

absorb the solar radiation. Gypsum board impregnated with PCM for thermal storage in a passive solar test room was investigated by Athienitis et al. [8]. It was shown that the utilization of the PCM gypsum board may reduce the maximum room temperature by about 4 °C during the daytime. The thermal performance of a randomly mixed PCMs and a laminated PCM-wall system was numerically evaluated by Kim and Darkwa [9], and a 20–50% heat flux enhancement was achieved by the laminated system. A full scale thermal testing of the latent heat storage in a wallboard was investigated by Scalat et al. [10]. The testing facility consists of two identical insulated adjacent rooms. Walls of one room were finished with ordinary gypsum board while the other room was finished with PCM impregnated gypsum board. It was seen that the latent heat release stabilized the room temperature for about 35.5 h. Heim and Clarke [11] studied the influence of the PCM on wallboard surface temperature through experimental work. The results indicated that there was no considerable reduction in the diurnal temperature fluctuations. Zhang et al. [12] presents the development of a thermally enhanced frame wall that reduces peak air conditioning demand in residential buildings using highly crystalline paraffin PCM. The average space-cooling load was reduced by 10.8% when the PCM was applied. The thermal dynamics of the PCM wallboard subjected to diurnal variations of a room temperature was investigated by Neepor [13]. It was indicated that the use of the PCM wallboard on the envelope of a building could not alter the net heat conduction in the envelope.

In this study, thermal analyses of a two-dimensional model for a common building brick with cylindrical holes containing phase change material is presented. The objective of the brick-PCM system is to reduce the heat flow from outdoor space by absorbing the heat gain in the brick before it reaches the indoor space during the daytime. At night, the stored heat is released to indoor and outdoor spaces.

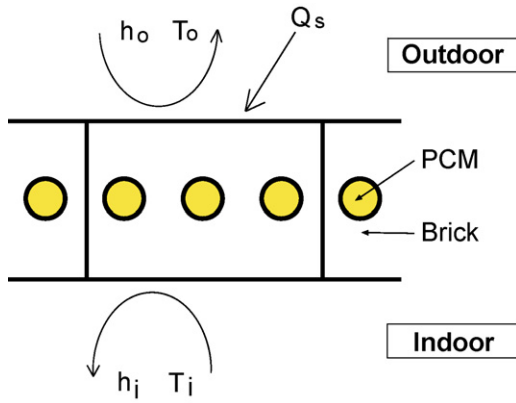


Fig. 2. Schematic representation of the brick-PCM system, and the boundary conditions.

## 2. Analysis

Fig. 2 depicts the geometry configuration of bricks containing three PCM cylinders. The wall consists of a 0.25 m × 0.15 m × 0.15 m of a horizontal brick with cylindrical holes. The diameter of the holes is 0.03 m. Four different cases are investigated, bricks with one, two, and three PCM cylinders, as well as, a brick without PCM. The outdoor surface of the wall is subjected to a time dependent solar radiation and forced convection boundary conditions, while the indoor surface is subjected to time independent free convection boundary condition. Three type of paraffin are examined as PCMs: *n*-octadecane, *n*-eicosane, and P116. The melting temperature of these PCMs is within the operating temperature of the brick-PCM system. Table 1 shows the thermophysical properties of the brick and PCMs [14]. Real radiation data was used to determine the boundary condition at the outdoor surface. Fig. 3 shows the measured average hourly variations of the ambient air temperature and global radiation (sum of direct and diffuse radiations) in Kuwait city [15]. The weather data is for month of June where the consumption of electrical power reaches its peak.

In general, the heat transfer in the wall is three dimensional in nature. Width of the walls is large comparing to its thickness,

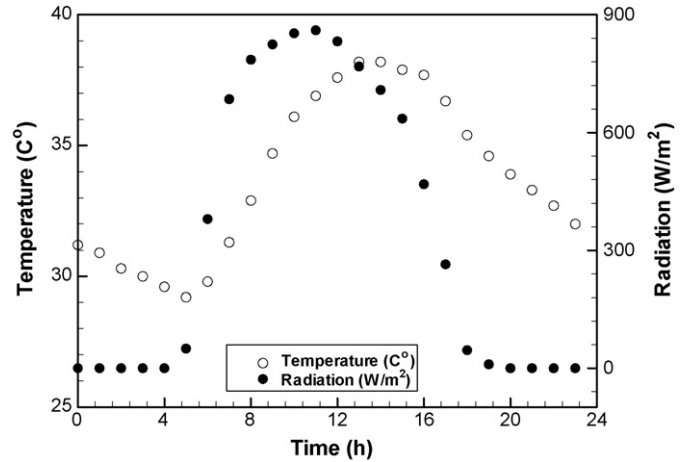


Fig. 3. Hourly variation of solar radiation and ambient temperature for the month of June.

and therefore, the walls' ends effect has a negligible effect on the heat transfer in the wall. Since there is symmetry in the problem, only a portion of the wall is considered in the analysis. Fig. 4 shows the computational domain with important geometric parameters and the finite element mesh. The properties of the brick are temperature independent, but they are dependent for the PCM. For simplicity, the thermal expansion of PCM and brick is not considered, and the effect of the natural convection of liquid PCM is not accounted in the computations. The natural convection of the liquid PCM is insignificant when the temperature different between initial condition and boundary is small [16], which is applicable in this research. Considering the above assumptions, one can write the two-dimensional heat conduction equations for the brick and the PCM:

$$\text{Brick : } (\rho C)_b \frac{\partial T_b}{\partial t} = k_b \left( \frac{\partial^2 T_b}{\partial x^2} + \frac{\partial^2 T_b}{\partial y^2} \right) \quad (1)$$

$$\text{PCM : } (\rho C)_{\text{PCM}} \frac{\partial T_{\text{PCM}}}{\partial t} = k_{\text{PCM}} \left( \frac{\partial^2 T_{\text{PCM}}}{\partial x^2} + \frac{\partial^2 T_{\text{PCM}}}{\partial y^2} \right) \quad (2)$$

By making the specific heat of the PCM function of temperature, the latent heat effect can be simulated. The definition of the PCM's specific heat is as follows:

$$(\rho C)_{\text{PCM}} = \begin{cases} (\rho C)_{\text{solid}} & T < T_m \\ \frac{(\rho C)_{\text{solid}} + (\rho C)_{\text{liquid}}}{2} + \frac{\rho_{\text{solid}} + \rho_{\text{liquid}}}{2} \left( \frac{\lambda}{\Delta T} \right) & T_m \leq T \leq T_m + \Delta T \\ (\rho C)_{\text{liquid}} & T > T_m + \Delta T \end{cases} \quad (3)$$

Table 1  
Thermophysical properties of the insulation and PCMs

Material	$T_m$ (°C)	$k$ (W/m °C)	$C$ (kJ/kg °C)	$\rho$ (kg/m <sup>3</sup> )	$\lambda$ (kJ/kg)
Bricks	–	0.7	0.84	1600	–
<i>n</i> -Octadecane	27	0.358 (solid), 0.148 (liquid)	1.934 (solid), 2.196 (liquid)	865 (solid), 780 (liquid)	243.5
<i>n</i> -Eicosane	37	0.15 (solid), 0.15 (liquid)	2.01 (solid), 2.04 (liquid)	778 (solid), 856 (liquid)	241.0
P116	47	0.24 (solid), 0.24 (liquid)	2.4 (solid), 1.9 (liquid)	830 (solid), 773 (liquid)	225.0

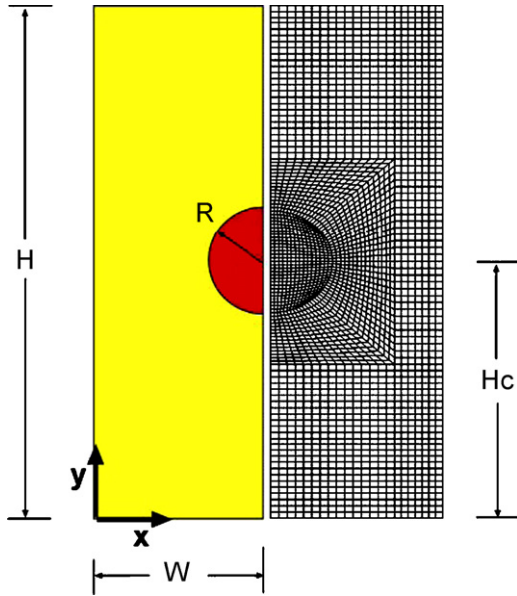


Fig. 4. The computational domain and the finite element mesh.

where  $\lambda$  is the latent heat of fusion,  $C$  the specific heat,  $T_m$  the PCM's melting temperature, and  $\Delta T$  is the phase change transition temperature,  $\Delta T = 1^\circ\text{C}$  [17]. The outdoor side of the wall is subjected to a time dependent radiation and forced convection boundary conditions, with  $h_o = 20 \text{ W/m}^\circ\text{C}$ . At the indoor surface, a free convection boundary condition, with  $h_i = 10 \text{ W/m}^\circ\text{C}$  and  $T_i = 23.5^\circ\text{C}$ , is imposed. The initial temperature of the domain is  $30^\circ\text{C}$ . The simulation was kept running until a periodic condition is established. The heat flow into the outdoor wall surface can be expressed as:

$$Q_{os} = Q_{conv} + Q_s - Q_{Correc} \quad (4)$$

or

$$Q_{os} = h_o A (T_o - T_{os}) + \alpha A q_s - \varepsilon A \sigma (T_o^4 - T_{surr}^4) \quad (5)$$

where  $T_o$  is the outdoor air temperature,  $T_{os}$  the outdoor surface temperature,  $\alpha$  the solar absorptivity of the wall surface, and  $q_s$  is the solar flux radiation. The last term in Eq. (5) represents the correction for the radiation heat transfer when the surrounding and the ambient temperatures are not equal [18]. It is assumed that these two temperatures are equal in this research. The heat flow into the outdoor surface can be expressed as:

$$Q_{os} = h_o A (T_s - T_{os}) \quad (6)$$

where  $T_s$  is the solar-air temperature, which is defined as:

$$T_s = T_o + \frac{\alpha q_s}{h_o} \quad (7)$$

The boundary condition at the indoor surface ( $y = 0$ ):

$$-k_b \frac{\partial T_b}{\partial y} = h_i (T_i - T_{is}) \quad (8)$$

The boundary condition at the outdoor surface ( $y = H$ ):

$$-k_b \frac{\partial T_b}{\partial y} = h_o (T_s - T_{os}) \quad (9)$$

where  $T_{is}$  is the indoor surface temperature.

At the brick/PCM interface:

$$k_{PCM} \frac{\partial T_{PCM}}{\partial n} = -k_b \frac{\partial T_b}{\partial n} \quad (10)$$

where  $n$  is the coordinate normal to the surface of the PCM cylinders.

Along the symmetry lines ( $x = 0$  and  $x = W$ ):

$$\frac{\partial T}{\partial x} \Big|_{x=0} = \frac{\partial T}{\partial x} \Big|_{x=W} = 0 \quad (11)$$

Heat flux at the indoor wall surface is calculated to study the effect of the PCM on the heat flow from outdoor space to indoor space, and it is defined as:

$$q_{is} = h_i (T_i - T_{is}) \quad (12)$$

The heat flux can be integrated to obtain the net heat flux for a specific period:

$$\bar{q}_{is} = \frac{1}{t} \int_{t_o}^{t+t_o} q_{is} dt \quad (13)$$

### 2.1. Finite element method

The finite element method is utilized to solve the problem. The pre-conditional generalized minimum residual (PGMR) solver [19] was used to solve a set of discretized equations for temperature field. A four-nodal quadrilateral element was used to mesh the computational domain. In order to ensure that a mesh independent solution is obtained, three finite elements models are examined with different number of elements: 2041, 2660, and 3103 elements. The results indicate that the difference between the second and third models is less than 1%. Hence, the second model is adopted for the numerical simulations. Fig. 4 shows the finite element mesh. At each time step, the convergence is evaluated, and the following condition is used to declare the convergence:

$$\frac{|T_{ij}^{m+1} - T_{ij}^m|}{|T_{ij}^m|} \leq 10^{-6} \quad (14)$$

The convergence monitor represents the sum of the changes of temperature calculated from the current ( $m + 1$ )th iteration and the previous ( $m$ )th iteration divided by the sum of current values. This ratio should be less than  $10^{-6}$  to declare convergence.

## 3. Results and discussion

### 3.1. The effect of PCM type

The effectiveness of the PCM-brick system is evaluated by comparing the heat flux at the indoor surface to the brick without PCM. The melting temperature of the three selected

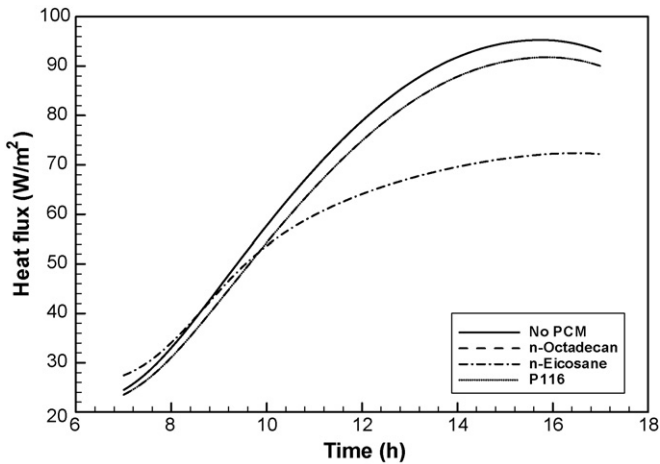


Fig. 5. Heat flux at the indoor surface for different type of the PCM.

PCMs is within the operating temperature of the system. P116, *n*-eicosane, and *n*-octadecane paraffin are investigated as potential PCMs. It is noticed from the temperature distributions of the brick without PCM that the temperature in the brick is between 25.7 °C and 55.6 °C. The melting temperature of the *n*-octadecane is closed to the lower limit of the operating temperature, while the melting temperature of the P116 wax is closed to the upper limit. The melting temperature of the *n*-eicosane is in the middle of the temperature limit. Fig. 5 shows the heat flux at the indoor surface for three types of PCMs, as well as brick without PCM, during the working hours, 7 a.m. to 5 p.m. Three PCM cylinders are used, and the PCM cylinders are located at the center of the brick,  $H_c = H/2$ . For the brick without PCM, the heat flux increases from 26 W/m<sup>2</sup> at 7 a.m. to reach its peak at 3 p.m. The figure indicates that the P116 and *n*-octadecane are ineffective in reducing the heat flux to the indoor space. The heat flux of the P116 and *n*-octadecane is very close to the brick without PCM. The low melting temperature of the *n*-octadecane made it in liquid phase all the time, while high melting temperature of the P117 made it in solid phase all the time. Therefore, the brick is not benefited from their high latent heat of fusion to reduce the heat flow to the indoor space. On the other hand, when *n*-eicosane is introduced, the rate of change of the heat flux is substantially reduced during the period from 10 a.m. to 5 p.m., with a maximum heat flux reduction of 24.2%. Notice that the difference between the examined cases is negligible during the period from 7 a.m. to 10 a.m. because the temperature of the brick is still below the PCM's melting temperature.

### 3.2. The effect of PCM quantity

The effect of the PCM quantity in the brick is studied. The objective of this study is to optimize the quantity of the PCM in the brick. Having a minimum quantity of the PCM is desirable to maintain the strength of the brick. A brick with one, two and three cylinders, as well as a brick without PCM is investigated. These cylinders are located at the centerline of the brick,  $H_c = H/2$ . Fig. 6 shows the heat flux at the indoor surface for the

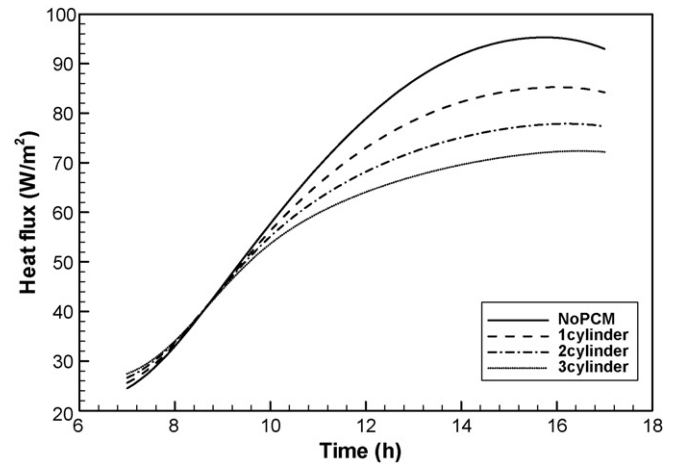


Fig. 6. Heat flux at the indoor surface for different number of the PCM cylinders.

four examined cases. The figure indicates that with only one cylinder the maximum heat flux is reduced by about 11.5%, and 17.9% with two cylinders. When three cylinders are used, the reduction reaches 24.2%. The total heat transfer at the indoor surface is heat gain that should be removed by HVAC system. The heat flux in Fig. 6 is integrated to obtain the total heat flux during the working hours. Fig. 7 shows the total heat flux reduction, which is the ratio of the heat flux of a brick with to a brick without PCM. The bar chart indicates that the total heat flux at the indoor space can be reduced by 17.55% when three PCM cylinders are introduced. Therefore, the energy cooling load can be decreased by this percentage when the PCM cylinders are incorporated into the bricks.

### 3.3. Temperature distributions

To aid in understanding the results in Figs. 6 and 7, temperature distributions in the brick with three PCM cylinders are presented at eight time steps during one day. Fig. 8 shows the temperature distributions at time = 0, 3, 6, 9, 12, 15, 18, and 21. In these figures, the upper surface is subjected to outdoor boundary

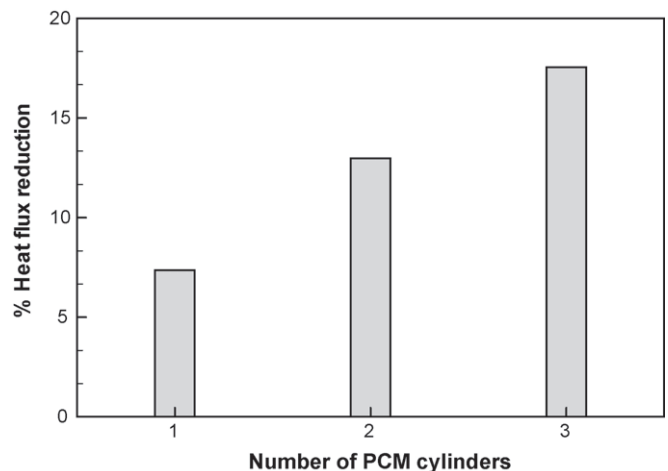


Fig. 7. Total heat flux reduction at the indoor surface for different number of PCM cylinders.

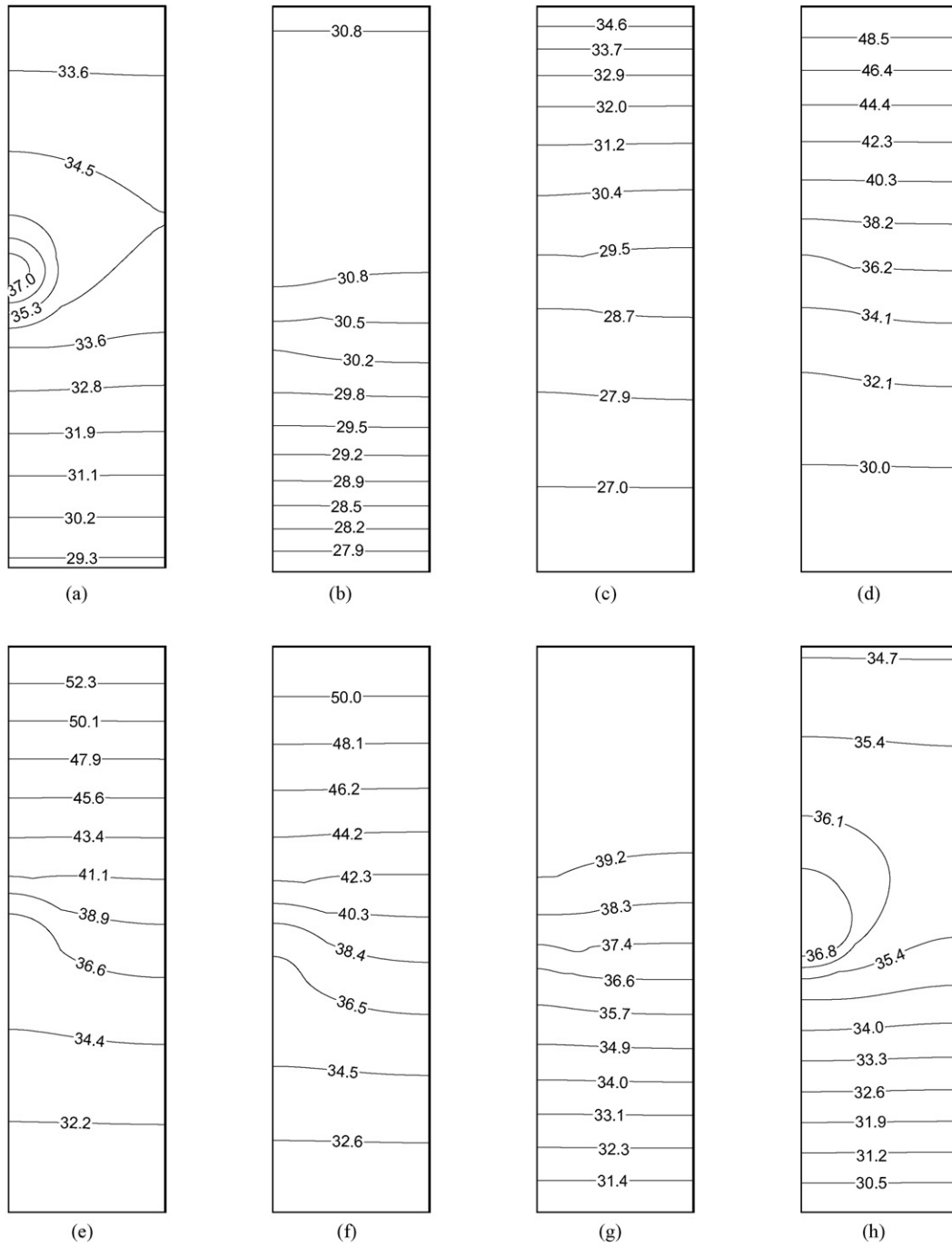


Fig. 8. Temperature contours in brick and PCM during 1 day, at time = (a) 0, (b) 3, (c) 6, (d) 9, (e) 12, (f) 15, (g) 18, and (h) 21.

condition, while the lower boundary is subjected to indoor boundary condition. The left and right sides are well insulated. The temperature contours indicate that the upper region is heavily varying between 30.8 °C and 55.6 °C, which are due to high variation of solar radiation and forced convection. On the other hand, the temperature variation at the lower region is not as high as the upper region, and the variation is between 25.7 °C and 32.5 °C. During the working hours, Fig. 8d–g indicates that the temperature at the center of the brick reaches 36.5 °C, which is the melting temperature of the PCM. The PCM starts to melt and absorb heat as it flows from the outdoor surface to indoor surface,

reducing the net heat flow to the indoor space. During the off working hours, Fig. 8h–c indicated that the temperature of the center of the brick is decreased below 36.5 °C, allowing the PCM to solidify. The net effect of this melting and solidification cycle of the PCM is a reduction in heat flow to the indoor space during the working hours.

### 3.4. The effect of PCM location in the brick

The location of the PCM cylinders in the brick is also investigated. The PCM cylinders are shifted from the centerline

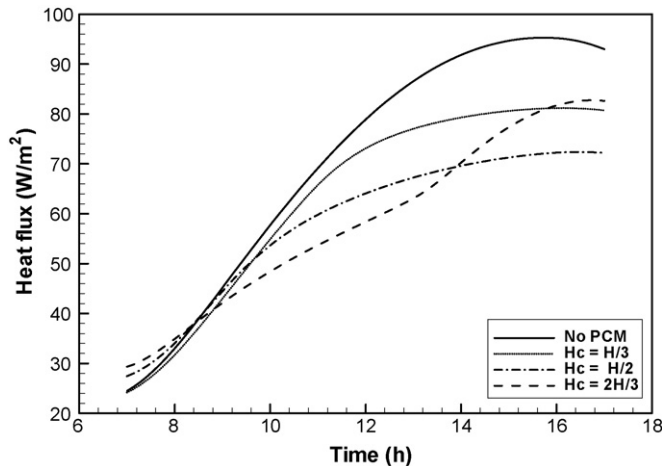


Fig. 9. Heat flux at the indoor surface for different locations of the PCM cylinders.

to a position close to the indoor and outdoor surfaces,  $H_c = H/3$  and  $H_c = 2H/3$ , respectively. The indoor and outdoor locations of the PCM cylinders are not preferred because it could make the strength of the brick weak.

Fig. 9 shows the heat flux at the indoor surface for the three examined locations, as well as the brick without PCM. The figure indicates that the thermal performance of the centerline case is better than the indoor position case. On the other hand, the outdoor cylinder position case has an initial better performance than the centerline case, up to 2 p.m. After this time, the heat flux increases rapidly. Being close to the outdoor surface, the PCM is subjected to a higher heat flow from the outdoor surface than other cases, leading to a fast melting rate. The sudden change in heat flux at 2 p.m. indicates that the PCM is completely melted. The heat flux during the working hours is integrated to evaluate the total heat flux at the indoor surface wall for the examined PCM locations. The result indicates that the total heat flux reduction for the outdoor and indoor PCM cylinders locations is 17.49% and 10.16%, respectively. For the centerline PCM location, the total heat flux reduction is 17.55%. Therefore, the centerline location of the PCM cylinders shows the best performance in term of thermal effectiveness, and also to maintain the strength of the bricks.

#### 4. Conclusion

A brick with cylindrical holes filled with PCM is investigated as a method for reducing the heat gain in buildings during the working hours. Weather data in a hot climate is used in the analyses. A parametric study is conducted to assess the effect of different design parameters, such as the PCM's quantity, type, and location in the brick. The results indicate that the heat gain is reduced when the PCM is incorporated in the brick, and increasing the quantity of the PCM has a positive

effect. The *n*-eicosane shows the best performance among the examined PCMs, and the centerline location of the PCM cylinders is the best in term of thermal effectiveness, and also to maintain the strength of the brick. The results indicate that the heat flux at the indoor space can be reduced by 17.55% when three PCM cylinders are introduced, placed at the centerline of the bricks.

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